Code No. 7951/CDE/N

FACULTY OF SCIENCE

M.Sc. (Previous) CDE Examinations, July / August 2019

Subject: Mathematics
Paper - I : Algebra

Time: 3 Hours **Max. Marks: 80**

Note : Answer any five questions from Part–A and all questions from Part–B. Each question carries 4 marks in Part–A and 12 marks in Part – B.

PART – A (5 x 4 = 20 Marks)

1. Define G-set. Show that a group G is a G-set under the action $a \ast x = a\, x\, a^{-1}$ for all $a, x \in G$.
2. If G is a group of order 108 then show that there exists a normal subgroup of order 27 or 9.
3. State and prove the fundamental theorem of homomorphism of rings.
4. Show that the ring of integers $\mathbb{Z}$ is a Euclidean domain.
5. If M is a simple R-module then prove that Hom_R (M, M) is a division ring.
6. Determine the minimal polynomials of $\sqrt{2}+5$ and $3\sqrt{2}+5$ over Q.
7. If $f(x) \in F[x]$ is an irreducible polynomial over a finite field F then prove that all the roots of $f(x)$ are distinct.
8. If F is a field of characteristic $\neq 2$ and $x^2 - a \in F[x]$ is an irreducible polynomial over F then prove that its Galois group is of order 2.

PART – B (5 x 12 = 60 marks)

9. a) If G is a solvable group then prove that every subgroup of G and every homomorphic image of a are solvable. Conversely if N is a normal subgroup of G such that N and G/N are solvable then G is solvable.

OR

b) State and prove fundamental theorem of finitely generated abelian groups.

10. a) If R is a commutative principal ideal domain with unity then prove that any non zero ideal P$\neq$ R is prime if and only if it is maximal.

OR

b) Let R = F[x] be a polynomial ring over a commutative integral domain F and f(x), g(x) $\neq$ 0 be polynomials in F[x] of degrees m and n respectively. Let $k = \max\{m-n+1,\ 0\}$ and a be the leading co-efficient of g(x) then prove that there exists unique Polynomials q(x) and r(x) in F[x] such that $a^k f(x) = q(x)g(x) + r(x) \quad where \quad r(x) = 0$ or $r(x)$ has degree less than the degree of $g(x)$.

11. a) (i) Prove that the submodules of the quotient module M/N are of the form U/N where U is a submodule of M containing N.

(ii) If A and B are R-submodules of R-modules M and N respectively then prove that $\frac{M \times N}{A \times B} \simeq \frac{M}{A} \times \frac{N}{B}$

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OR

b) If E is an algebraic extension of a field F and σ : F→L is an embedding of F into algebraically closed field L then prove that σ can be extended to an embedding ¦ $: E \rightarrow L$

12. a) State and prove fundamental theorem of Galois theory.

OR

b) Prove that f(x) $\in$ F[x] is solvable by radicals over F if and only if its splitting field E over F has solvable Galois group.

13. a) Prove that any two composition series of a finite group are equivalent.

OR

b) If E is a finite separable extension of a field F then prove that E is a simple extension of F.

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Code No. 7956/CDE/O/BL

FACULTY OF SCIENCE

M.Sc. (Previous) CDE Examinations, July/August 2019

Subject: Mathematics
Paper - I : Algebra

Time: 3 Hours **Max. Marks: 100**

Note: Answer any five from the following questions.
All questions carry equal marks.

1. a) If ¤ is a homomorphism of G onto $\overline{G}$ with kernel K then prove that $\frac{G}{K} \simeq \overline{G}$

   b) State and Prove Sylow's first theorem.

2. If G is a finite group then prove that $c_a = \frac{O(G)}{O(N(a))}$

3. If O(G) = p^2 where p is a prime prove that G is abelian.

4. Prove that every integral domain can be embedded in a field.

5. (a) If R is a Euclidean ring and a, b $\in$ R where b≠0 is not a unit is R then prove that d(a) < d(b)
   (b) If f(x) and g(x) are primitive polynomials then prove that f(x) g(x) is also a primitive polynomial.

6. (a) If a, b $\in$ K are algebraic over F then prove that a±b, ab and $\frac{a}{b}$ where b ≠0 are all algebraic over F.
   (b) If f[x] $\in$ F[x] then prove that there is a finite extension E of F in which f(x) has a root, and [E:F] $\leq$ deg. f(x)

7. If F is of characteristic 0 and if a, b are algebraic over F then prove that there exists an element c $\in$ F (a, b) such that F(a,b) = F(c).

8. Let K be the splitting field of f(x) $\in$ F[x] and G(K,F) be its Galois group. Let T be any subfield of K containing F onto the set of sub groups of G(K, F) such that
   (i) $T = K_{G(K,T)}$
   (ii) $H = G(K, K_H)$
   (iii) $[K:T] = 0\,(G(K,T))$ and [T: F] = index of G(K, T) in G(K, F)
   (iv) T is a normal extension of F if and only if G(K, T) is a normal subgroup of G(K, F)
   (v) If T is a normal extension of F then G(T, F) is isomorphic to G(K, F)/ G(K, T)

9. a) State and prove Schrerier's theorem.
   b) If L is a complemented modular Lattice and a, b $\in$L with a $\geq$ b then prove that there is an element $b_1 \in$L such that $b_1 \leq$ a, b∨b_1 = a and b∧b_1 = 0

10. Prove that Boolean algebra and Boolean ring with identity are equivalent.

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Code No. 7969/CDE

FACULTY OF SCIENCE

M.Sc. (Previous) CDE Examination, July/August 2019

Subject : Statistics

Paper – I : Mathematical Analysis & Linear Algebra

Time : 3 Hours **Max. Marks: 80**

Note: Answer any five from Part – A and answer all the questions from Part – B using internal choice.

PART – A (5x4 = 20 Marks)

1. Examine the function $f(x) = x^2 \cos(1/x)$ if $x \neq 0$
   $= 0$ if $x = 0$
   is of bounded variation on [0,1].
2. Define Riemann-stieltjes integral.
3. State chain rules for Jacobians.
4. State the properties of line integrals.
5. Define Moore-penrose Inverse.
6. State Hermite Form of a matrix and write an example.
7. Find the matrices for the Quadratic form $2x_1^2 + 8x_1x_2 + 6x_2^2$.
8. Define index and signature of a Quadratic form.

PART – B (5x12=60 Marks)

9. (a) i) State and prove the First Mean value Theorem for R-S integrals.
   ii) Investigate the nature of critical points of $f(x,y) = x^4+y^4-x^2-y^2+1$.

OR

   (b) i) Define Jarobian function of n variables.
   ii) If $x^2+y^2+u^2-v^2=0$ and $uv+xy=0$, find the Jarobian of transformation and the derivatives $\frac{\partial u}{\partial x}$ and $\frac{\partial g}{\partial y}$.

10. (a) Define Power series, Taylor's series, Laurent's series zeros and poles.

OR

   (b) State and prove the Cauchy-residue Theorem.

11. (a) If A^+ is the Moore penrose in verse of A then show that
   (i) $(A^+)^+=A$ (ii) $P(A) = P(A^+)$

OR

   (b) A solution to the system AX=b exists off the rank of the coefficient matrix A is equal to the rank of the augmented matrix B.

12. (a) Show that the ranges of values of two congruent Quadratic Forms (Q.F's) are same.

OR

   (b) Find the rank, index, canonical form to the Quadratic form
   $x_1^2 + 2x_2^2 + 3x_3^2 + 2x_2x_3 + 2x_1x_3 + 2x_1x_2$.

13. (a) If $A=(a_{ij})$ is positive definite then show that $|A| \leq a_{11}\, a_{22} \ldots a_{mm.}$

OR

   (b) State and prove Cauchy-Schwartz and Hadamard inequalities.

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Code No. 7952/CDE/N

FACULTY OF SCIENCE

M.Sc. (Previous) CDE Examinations, July / August 2019

Subject: Mathematics

Paper: II Real Analysis

Time: 3 Hours **Max. Marks: 80**

Note : Answer any five questions from Part–A and all questions from Part–B. Each question carries 4 marks in Part–A and 12 marks in Part – B.

PART – A (5 x 4 = 20 Marks)
(Short Answer Type)

1. Prove that every closed subset of a compact set is compact.
2. Prove that a set E is open if and only if its complement is closed.
3. Prove that a continuous mapping defined on a compact metric space is uniformly continuous.
4. If f is monotonic on (a, b) then prove that the set of points (a,b) at which f is discontinuous is at most countable.
5. With usual notations prove that $\int_{\underline{a}}^{b} fd\sim \;\le\; \int_{a}^{\bar{b}} fd\sim$
6. If f is continuous on [a, b] then prove that $f \in R(\sim)$ on [a, b]
7. Give an example of a series of continuous functions whose sum function is discontinuous.
8. State and prove contraction principle.

PART – B (5 x 12 = 60 marks)
(Essay Answer Type)

9\. a) Define compact set. Prove every K-call is compact.

OR

b) If E is a subset of R^k then prove that the following are equivalent.
(i) E is closed and bounded.
(ii) E is compact.
(iii) Every infinite subset of E has a limit point in E.

10\. a) Suppose (X, d) is a metric space and f, g are real valued functions defined on ECX. Suppose 'P' is a limit point of E. let $\lim\limits_{x\to p} f(x) = q$ and $\lim\limits_{x\to p} g(x) = q'$ then

prove (i) $\lim\limits_{x\to p} (f+g)(x) = q + q'$

(ii) $\lim\limits_{x\to p} (fg)(x) = qq'$

OR

b) If f is monotonically increasing on (a, b) then prove $f(c+)$ and $f(c-)$exists at every point c of (a, b) in fact $\sup\limits_{a<t<c} f(t) = f(c-) \le f(c) \le f(c+) = \inf\limits_{c<t<b} f(t)$ further if a<c<d<b then prove $f(c+) \quad f(c-)$

11\. a) suppose F and G are differentiable functions on [a, b], $F' = f \in R$ and $G' = g \in R$.

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Then prove $\int_a^b F(x)g(x)dx = F(b)G(b) - F(a)G(a) - \int_a^b f(x)G(x)dx$

OR

b) if ¥′ is continuous on [e, b] then prove ¥ is rectifiable and $\Lambda(¥) = \int_a^b |¥'(t)|\ dt$

12. a) Suppose $f_n \to f$ .uniformly on a set E in a metric space let x be a limit point of E, and suppose that $\lim_{t\to x} f_n(t) = A_n$ (n = 1, 2, 3,……). Then prove $\{A_n\}$ converges and $\lim_{t\to x} f(t) = \lim_{n\to\infty} A_n$

OR

b) Show that there exists a real continuous functions on the real line which is no where differentiable.

13. a) State and prove Weierstrass approximation theorem.

OR

b) (i) If $A \in L(\mathbb{R}^n, \mathbb{R}^m)$ then prove $\|A\| < \infty$ and A is a uniformly continuous mapping of $\mathbb{R}^n$ into $\mathbb{R}^m$.

(ii) If $A \in L(\mathbb{R}^n, \mathbb{R}^m)$ and $B \in (\mathbb{R}^m, \mathbb{R}^k)$ then prove $\|BA\| \le \|B\|\ \|A\|$

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Code No. 7957/CDE

FACULTY OF SCIENCE

M.Sc. (Previous) CDE Examinations, July 2019

Subject: Mathematics

Paper: II Analysis

Time: 3 Hours **Max. Marks: 100**

Note: Answer any five of the following questions, choosing at least two from each part. All questions carry equal marks.

1. a) Suppose A is the set of all sequences whose elements are the digits 0 and 1. Prove that the set A is uncountable.

   b) Suppose $Y \subset X$. Prove that a subset E of y is open relative to Y if and only if $E = Y \cap G$ for some open subset G of X.

2. a) If $\{K_\alpha\}$ is a collection of compact subsets of a metric space X is non-empty. Prove that , the intersection of every finite subcollection of $\{K_\alpha\}$ is non empty. Prove that $\cap K_\alpha$ is no empty.

   b) Explain the construction of Cantor's set.

3. a) Prove that the subsequential limits of a sequence $\{p_n\}$ in a metric space form a closed subset of X.

   b) Prove that the Cauchy product of an absolutely convergent series and a convergent series is convergent and converges to the product of their sums.

4. a) Suppose X, Y, Z are metric spaces, $E \subset X$, f maps E into Y of maps the range of f, f(E) into Z and h is the mapping of E into Z defined by $h(x) = g(f(x))$ $(x \in E)$. If f is continuous at a point $P \in E$ and g is continuous at the point f(p) them prove that h is continuous of P.

   b) Prove that continuous image of a connected set is connected.

5. (a) Suppose f is continuous on [a, b]. Prove that $f \in R(\alpha)$ on [a, b].

   (b) If $f \in R(\alpha)$ on [a,b] and if a < c < b. Prove that $f \in R(\alpha)$ on [a,c] and on [c,b]. Also Prove that

$$\int_a^b f \, d\sim = \int_a^c f \, d\sim + \int_c^b f \, d\sim$$

6. (a) Suppose $C_n \geq 0$ for n = 1, 2,….. and ΣC_n converges, $\{s_n\}$is a sequence of distinct points in (a, b) and

$$\sim(x) = \sum_{n=1}^{\infty} C_n I(x - s_n)$$

   Suppose f is continuous on [a, b] prove that $\int_a^b f d\sim = \sum_{n=1}^{\infty} C_n f(s_n)$

   (b) State and prove the fundamental theorem of Calculus.

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7. (a) State and prove the Weistrass-M test for uniform convergence of a series of functions.
   (b) Suppose f_n f uniformly on a set E in a metric space. Let x be a limit point of E and suppose that

   $\lim\limits_{t \to n} f(t) = A_n \, (n = 1,2,3,.....)$ prove that $\{A_n\}$ converges and $\lim\limits_{n \to \infty} A_n = \lim\limits_{t \to x} f(t)$

8. a) Suppose K is a compact metric space and $f_n \in ¸(k)$ from n = 1, 2, 3, ….. and if $\{f_n\}$ converges uniformly on K then prove that $\{f_n\}$ is equicontinuous on K.
   b) If f is a continuous complex function on [a, b] prove that there exists a sequence of polynomials P_n such that $\lim\limits_{n \to \infty} P_n(x) = f(x)$ uniformly on [a,b]

9. a) suppose be additive regular, non-negative and finite on ξ and ECR^p. Define *(E) the outer measure of E. Also prove that
   i) *(A) = (A) for energy A $\in \xi$
   ii) If E = $\mathop{U}\limits_{n=1}^{\infty} E_n$ then μ^*(E) $\leq \sum\limits_{n=1}^{\infty} «^*(E_n)$
   b) Define a measurable function. Suppose $\{f_n\}$ is a sequence of measurable functions on X such that for each $x \in$ X define
   $g(x) = \sup f_n(x) (n = 1,2,3.....)$ *and* $h(x) = \lim\limits_{n \to \sim} \sup f_n(x)$ prove that g and h are measurable functions on X.

10. a) Suppose f is measurable and non negative on X. For A$\in \mathcal{M}$ define
    ϕ(A) = $\int\limits_A fd«$ Prove that ϕ is countably additive on $\mathcal{M}$.
    b) Let $\{\phi_n\}$ be a complete orthonormal set. If $f \in L^2(«)$ and if $f \sim \sum\limits_{n=1}^{\infty} c_n$ ¤$_n$ then prove that $\int\limits_X |f|^2 du = \sum\limits_{n=1}^{\infty} |c_n|^2$

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Code No. 7970/CDE

FACULTY OF SCIENCE

M.Sc. (Previous) CDE Examination, July / August 2019

Subject : Statistics

Paper – II : Probability Theory

Time : 3 Hours **Max. Marks: 80**

Note: Answer any five questions from Part – A and all questions from Part – B. Each question Carries 4 marks in Part – A and 12 marks in Part – B.

PART – A (5x4 = 20 Marks)
(Short Answer Type)

1. State and prove Bayes theorem. What are its applications?
2. Prove that if $E(x^2) < \infty$ then $v(x)=v[E(x/y)] + e[v(x/y)]$.
3. Define characteristic function and state Levy-cramer continuity theorem.
4. Define : (i) convergence in probability and (ii) convergence in distribution. Disucss the implications and counter implications.
5. State and prove Demoiver – Laplace CLT.
6. State kolmogorov's inequality. Mention its applications.
7. Define first entrance probability and first return probability.
8. Define n^{th} order transition probability and explain gamblers ruin problem.

PART – B (5x12=60 Marks)
(Essay Answer Type)

9. (a) i) State and prove chebychev's inequality and
ii) State and prove Markov inequality.

OR

(b) i) Prove that if x is a r.v then $\frac{1}{x}$ is also a r.v.

ii) If $f(x,y)=\begin{cases} 6xy(2-x-y); 0 < x, y < 1 \\ 0 ; 0.w \end{cases}$.

Find $E(x/y)$ and $V(x/y)$.

10. (a) i) Prove $x_n \xrightarrow{L} x \Rightarrow g(x_n) \xrightarrow{l} g(x)$.

ii) Define almost sure convergence and prove that if $x_n \xrightarrow{a.s} x \Rightarrow x_n \xrightarrow{p} x$.

OR

(b) i) Obtain the characteristic function of a r.v $x \sim N(\mu\ \ \sigma^2)$.
ii) Find the density function of a r.v x whose characteristic function $ø_x(t)=e^{-|t|}$.

11. (a) State and prove Lindberg – Levy's CLT.

OR

(b) i) State and prove kolmogorov's SLLN for a sequence of independent r.v.'s.

ii) Let $\{x_n\}$ be a sequence of independent r.v,'s with $P(x_n= \pm 2^n)=\frac{1}{2}$. Examine SLLN holds for this sequence.

12. (a) If $\{x_n; n \geq 0\}$ is a Markov chain defined on the state space {0, 1, 2, 3} with TPM

$$P=\begin{bmatrix} 1/3 & 2/3 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1/4 & 0 & 3/4 & 0 \\ 0 & 0 & 1/5 & 4/5 \end{bmatrix}$$

(i) Classify the states of the chain.
(ii) Find $P[x_1=1, x_2=2]$ and
(iii) Compute $P_{11}^{(2)}, P_{21}^{(2)}$.

OR

(b) Derive a sufficient condition to examine whether a state of a markov chain is persistant or transient.

13. (a) i) State and prove chebycher's WLLN.
ii) For the sequence of r.v.'s such that $P[x_n = \pm 2^n] = \frac{1}{2}$ n≥1, examine whether WLLN holds.

(b) i) Obtain the MGF of poisson r.v with parameter λ.
ii) Show that MGF does not exists for a Cauchy r.v.

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Code No. 7953/CDE/N

FACULTY OF SCIENCE

M.Sc. (Previous) CDE Examinations, July / August 2019

Subject: Mathematics

Paper- III : Topology & Functional Analysis

Time: 3 Hours **Max. Marks: 80**

Note : Answer any five questions from Part–A and all questions from Part–B. Each question carries 4 marks in Part–A and 12 marks in Part – B.

PART – A (5 x 4 = 20 Marks)

1. Let X be a non-empty set and consider the class of subsets of X consisting of the empty set ϕ and all sets whose complements are countable. Is this a topology on X?
2. Show that a topological space is compact if and only if every class of closed sets with the finite intersection property has non-empty intersection.
3. Show that in a T_2-space, any point and disjoint compact subspace can be separated by open sets, in the sense that they have disjoint neighbourhoods.
4. Show that the components of a totally disconnected space are its points.
5. Let T be a bounded linear operator, then prove that
   (i) $x_n \to x \;\; implies \; Tx_n \to Tx$.
   (ii) N(T) is closed
6. Show that dual space of $\mathbb{R}^n$ is $\mathbb{R}^n$.
7. Let H be a Hilbert space and $T : H \to H$ be a bounded linear operator, then prove that if T is self-adjoint, $\langle Tx, x \rangle$ is real for all $x \in H$.
8. Let H_1, H_2 be Hilbert spaces and $S : H_1 \to H_2$ and $T : H_1 \to H_2$ be bounded linear operators and α is any scalar. Then prove that
   (i) $\langle T^* y, x \rangle = \langle y, Tx \rangle \quad x \in H_1, y \in H_2$
   (ii) $(S+T)^* = S^* + T^*$
   (iii) $(\sim T)^* = \overline{\sim} T^*$

PART – B (5 x 12 = 60 marks)

9. a) (i) Show that a metric space is compact if and only if it is complete and totally bounded.
   (ii) Show that a closed subspace of a complete metric space in compact if and only if it is totally bounded.

OR

   b) State and prove Ascoli's Theorem.

10. a) (i) Show that every compact Hausdorff space is normal.
   (ii) Let X be a T_1 space, and show that X is normal if and only if each neighborhood of a closed set F contains the closure of some neighborhood of F.

OR

   b) (i) Let X be a Hausdorff space. Prove that if X has an open base whose sets are also closed, then X is totally bounded.
   (ii) Let X be a compact T_2-space. Then prove that X is totally disconnected if and only if it has an open base whose sets are also closed.

11. a) Prove that is Y is a Banach space then B(X, Y) is a Banach space.

OR

b) Show that dual space of l^p is l^q where $1 < p < \infty$ and q is conjugate of p,

(that is, $\frac{1}{p}+\frac{1}{q}=1$).

12. a) Show that every bounded linear functional f on a Hilbert space H can be represented in terms of the inner product, namely, $f(x)=\langle x, z\rangle$ where z depends on f, is uniquely determined by f and has norm, $\|z\|=\|f\|$.

OR

b) State and prove Ries z representation theorem for Hilbert spaces.

13. a) State and prove Bessel's inequality.

OR

b) (i) Define direct Sum in vector spaces.

(ii) Let Y be any closed subspace of a Hilbert space H. Then Prove that $= Y \oplus Z$ where $Z = Y^{\perp}$.

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Code No. 7958/CDE/BL

FACULTY OF SCIENCE

M.Sc. (Previous) CDE Examinations, July/August 2019

Subject: Mathematics

Paper- III : Differential Equations & Complex Analysis

Time: 3 Hours **Max. Marks: 100**

Note: Answer any five of the following questions, choosing at least two from each part. All questions carry equal marks.

1. a) Prove that $\int_{-1}^{1} P_m(x)P_n(x)dx = 0 \; if \; m \neq n$

   b) Evaluate $\int x^4 J_1(x)dx$.

2. a) Show that $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}\left(x^2-1\right)^n$.

   b) Solve in series $x^2y'' + xy' + \left(x^2 - n^2\right)y = 0.$

3. a) State and prove the necessary and sufficient condition for the equation $Pdx + Qdy + Rdz = 0$ to be exact.

   b) Find the curves represented by the solution of $ydx + (z-y)dy + xdz = 0.$

4. a) Explain Charpit's method and hence solve $\left(p^2+q^2\right)y = qz$.

   b) Find the surface satisfying $r = 6x + 2$ and touching $z = x^3 + y^3$ along its section by the plane $x + y + 1 = 0$.

5. (a) Solve $\left(D^2 - 6DD' + 9D'^2\right)Z = 12x^2 + 36xy$.

   (b) Explain Monge's method of integrating
   Rr+Ss+Tt = v

6. (a) Derive necessary condition for f(z) to be analytic in Polar coordinates.

   (b) Find the radius of convergence of the series.

   (i) $\sum_{n=0}^{\infty} n^n z^n$ (ii) $\sum_{n=0}^{\infty} \frac{z^n}{n!}$ (iii) $\sum_{n=1}^{\infty} \frac{z^{2n}}{2^n}$

7. a) Show that if $f(z) = u + iv$ is analytic then the curves $u(x,y) = c_1$ and $v(x,y) = c_2$ form an orthogonal system of curves.

   b) Show that the mapping $w = \frac{1+z}{1-z}$ maps $|\, z \,| \leq 1$ onto the half plane $\mathrm{Re}(w) \geq 0$

8. a) State and prove Cauchy's integral formula.

   b) Suppose G is an open set and $f : G \to C$ is differentiable then prove that f is analytic in G.

9. a) Discuss various types of singularities.

   b) Using Residues show that $\int_{0}^{2\pi} \frac{d\theta}{1+a\cos\theta} = \frac{2\pi}{\sqrt{1-a^{2}}}$

10. a) State and prove argument principle.

    b) State and prove maximum modulus theorem.

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Code No. 7971/CDE

FACULTY OF SCIENCE

M.Sc. (Previous) CDE Examination, July/August 2019

Subject : Statistics

Paper – III : Distribution Theory & Multivariate Analysis

Time : 3 Hours **Max. Marks: 80**

Note : Answer any five questions from Part–A and all questions from Part–B. Each question carries 4 marks in Part–A and 12 marks in Part – B.

PART – A (5x4 = 20 Marks)
(Short Answer Type)

1. Explain Pareto distribution and find its mean and variance.
2. Define lognormal distribution and state its properties.
3. If X, Y are independent U(0,1), then find p.d.f. of XY.
4. Find the mean and variance of the normal distribution truncated on left side at x=A. Give an illustration.
5. If a multivariate normal vector is partitioned into two sub vectors, which are uncorrelated, then show that the two sub vectors are independent multivariate normal vectors.
6. define Hotelling's T^2 statistic and Mahalnobi's D^2 statistic and establish their relationship.
7. Define principal components and state its importance and applications.
8. Describe factor analysis and give orthogonal factor model.

PART – B (5x12=60 Marks)
(Essay Answer Type)

9. (a) i) Explain gamma distribution and obtain its moment generating function.
   ii) Derive the moment generating function of exponential distribution and hence obtain it's mean and variance.

OR

   (b) i) Derive the characteristic function of Beta distribution of first kind and obtain its characteristic function. Hence find it's mean and variance.
   ii) Define the joint, marginal and conditional probability density functions. Obtain the characteristic function of Laplace distribution and hence find mean and variance.

10. (a) i) Define the compound Poisson distributions and also derive the mean and variance of the distribution.
   ii) Define truncated exponential distribution. Derive its mean and variance.

OR

   (b) i) Define order statistics. Obtain the marginal distribution of r^{th} order statistic.
   ii) Derive the distribution of ¡^{2}. Find its mean and variance.

11. (a) Define multivariate normal distribution. Obtain the maximum likelihood estimate of parameters μ and Σ.

OR

   (b) Prove that the conditional distribution obtained from the multivariate normal distribution is also multivariate normal.

12. (a) Describe the concept of discriminant analysis. Derive linear discriminant function and hence describe the classification between two multivariate populations.

OR

(b) Explain the objective of cluster analysis. Derive the hierarchical clustering methods namely single linkage, complete linkage and average linkage.

13. (a) i) Discuss a test procedure for testing the equality of mean vectors of two multivariate normal populations having equal dispersion matrix.

ii) Explain the method of computing the principal components based on the given sample dispersion matrix S.

OR

(b) If $\overline{X}$ and S^2 are the sample mean and sample variance based on a random sample from a normal population, then derive their sampling distributions and show that they are independent.

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Code No. 7955/CDE/N

FACULTY OF SCIENCE

M.Sc. (previous) CDE Examinations, July / August 2019

Subject: Mathematics

Paper: V Mathematical Methods

Time: 3 Hours **Max. Marks: 80**

Note: Answer any five from the following questions

PART – A (5 x 4 = 20 Marks)

1. Explain Frobenius method of solving .
   $\sim(x)y'' + \quad (x)y' + ¥(x)y = 0$
2. Show that $J_{1/2}(x) = \sqrt{\frac{2}{®x}} \sin x$
3. Show that the functions $x_1(t) = t^2, x_2(t) = t\,|\,t\,|$ are linearly independent on $-\infty < t < \infty$
4. Find e e^{tA} when $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$
5. Solve the IVP $x' = -x, \quad x(0) = 0, \quad t \geq 0$ using successive approximations method.
6. Show that the function of $f(t,x) = e^t x^{1/2}$ does not satisfy Lipschitz condition on S=$\{(t,x) : |\,t\,| \leq 2, \; |\,x \leq 1|\}$
7. Solve $\sqrt{p} + \sqrt{q} = 2x$
8. Explain the method of solving $f(p, \; q, z) = 0$

PART – B (5 x 12 = 60 marks)

9. a) Solve in series $2x^2 y'' + \left(x^2 - x\right) y' + y = 0$

OR

b) State and prove Rodrigue's formula for $P_n(x)$

10. a) Let $¤_1(t)$ a solution of $L(x) = x'' + b_1(t)x' + b_2(t)x = 0$ where b1, b2 are continuous on an interval I, and let $¤_1(t) \neq 0$ on I. Let $t_0 \in I$ then show that the second solution $¤_2(t) = ¤_1 \int_{t_o}^{t} \frac{1}{¤_1{}^2(s)} \exp\left[-\int_{t_o}^{t} b_1(u)du\right] ds$

OR

b) State and prove Abel's Formula.

11. a) State and prove existence and uniqueness theorem.

OR

b) Let f be continuous function defined on the rectangle $R : |t - t_o| \leq a, |x - x_o| \leq b$, a, b >0 then show that a function ¤ is a solution of the IVP $x' = f(t,x), \quad x(t_o) = x_o$ on an interval I containing the point t_o if and only if it is solution of the integral equation $x(t) = x_0 + \int_{t_0}^{t} f(s, x(s))ds$

12. a) Solve: $2zx - px^2 - 2qxy + pq = 0$ using Charpit's method.

OR

b) Solve one dimensional wave equation by separation of variable method.

13. a) solve: $\left(D^2 + DD' - 6D'^2\right)z = yCosx$

OR

b) Solve: (i) $\left(x^2 - yz\right)p + \left(y^2 - zx\right)q = z^2 - xy$

(ii) Form a partial differential Equation by eliminating arbitrary function f from f(x+y+z , x^2 + y^2 + z^2)=0

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Code No. 7973/CDE

FACULTY OF SCIENCE

M.Sc. (Previous-Practical) CDE Examination, July / August 2019

Subject : Statistics

Paper – I: Linear Algebra, Distribution Theory & Multivariate Analysis

Time : 3 Hours **Max. Marks: 100**

Note: Answer any THREE questions. All questions carry equal marks.

1. Find the generalized inverse of the following matrix:

$$\begin{bmatrix} -2 & 6 & 4 \\ 1 & -3 & 2 \\ 1 & 5 & 2 \end{bmatrix}$$

2. Obtain the spectral decomposition of the following matrix:

$$\begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

3. Fit an appropriate distribution and test for its goodness of fit to the following data which gives the number of dodders in a sample of clover seeds.

No.of dodders(x)	0	1	2	3	4	5	6	7	8
Observed frequency(f)	56	156	132	92	37	22	4	0	1

4. Fit a normal distribution to the following data and test for its goodness of fit.

Class	60-65	65-70	70-75	75-80	80-85	85-90	90-95	95-100
Frequency	3	21	150	335	326	135	26	4

5. In a diabetic centre the fasting blood sugar levels of a group of patients are recorded two times, one before the treatment (X1) and another after the treatment (X2).

X_1	X_2	X_1	X_2
172	174	329	310
222	200	314	303
110	206	228	343
233	218	215	311
366	148	153	215
181	366	179	303
185	205	156	130

Carry out the principle component analysis for the above data and obtain first principal component and its variance.

6. The following is sample correlations for five stocks:

$$D = \begin{bmatrix} 1 & & & & \\ .58 & 1 & & & \\ .51 & .60 & 1 & & \\ .39 & .39 & .44 & 1 & \\ .46 & .32 & .43 & .52 & 1 \end{bmatrix}$$

Treating the sample correlation coefficients as similarity measures, cluster analyse the stocks using the single linkage and complete linkage methods.

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